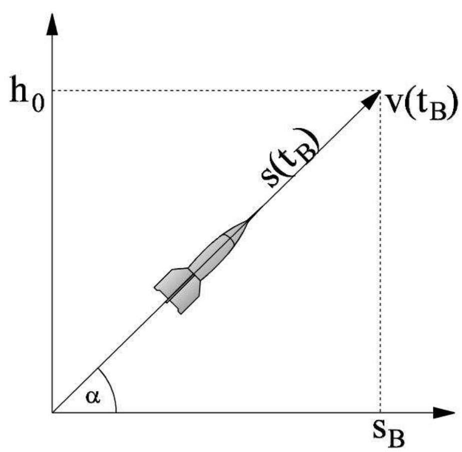
Parabolic trajectory of a rocket in a uniform gravitational field in absence of any other forces Example DoubleShot

The horizontal distance s of a parabolic trajectory in a uniform gravitational field and in the absence of other forces is given by

(0.1) $$s = \frac{v_0 \cdot \cos(\alpha)}{g} \cdot \left(v_0 \cdot \sin(\alpha) + \sqrt{v_0^2 \cdot \sin^2(\alpha) + 2 \cdot g \cdot h_0} \right)$$

with the initial velocity v_0, the initial angle α, the initial height h_0 and the acceleration of gravity g.

The rocket shall be launched from the completely flat surface with the initial angle α. We further assume the trajectory till burn-out $s(t_B)$ is a straight line:



We get

(0.2) $$\sin(\alpha) \cdot s(t_B) = h_0$$

(0.3) $$\cos(\alpha) \cdot s(t_B) = s_B$$

The initial velocity v_0 is the burn-out velocity $v(t_B)$, and thus the total horizontal traveled distance s_t of the rocket is:

(0.4) $$s_t = \frac{v(t_B) \cdot \cos(\alpha)}{g} \cdot \left(v(t_B) \cdot \sin(\alpha) + \sqrt{v(t_B)^2 \cdot \sin^2(\alpha) + 2 \cdot g \cdot h_0} \right) + s_b$$

The burn-out velocity $v(t_B)$ is given by the rocket equation

(0.5) $v(t_B) = v_{rel} \cdot \ln\left(\frac{m_A}{m_E}\right) - g \cdot t_B$

where

m_A = Initial total mass of the rocket
m_E = Final total mass of the rocket
t_B = Burn time
v_{rel} = Effective exhaust velocity
g = Gravitational acceleration

The straight line trajectory $s(t_B)$ is given by

(0.6) $s(t_B) = \int \left(v_{rel} \cdot \ln\left(\frac{m_A}{m_E}\right) - g \cdot t_B \right) dt_B = v_{rel} \cdot t_B - \frac{v_{rel} \cdot m_E \cdot t_B \cdot \ln\left(\frac{m_A}{m_E}\right)}{m_A - m_E} - \frac{g \cdot t_B^2}{2}$

<u>DoubleShot vehicle data</u>:

$m_P = 90.12kg$
$m_A = 34.94kg + 1.39kg + 2 \cdot 45.06kg = 126.45kg$
(Remark: Delay plug mass is considered as part of vehicle dry mass)

$m_E = m_A - m_p = 126.45kg - 90.12kg = 36.33kg$

$t_B = 13.4s$
(Remark: DoubleShot rocket motor considered as a 1-phase-motor)

$I_{sp} = 130s$

Launch angle $\alpha = 85^\circ$
(Remark: 0m level evaluation between launch and landing point)

$v_{rel} = I_{sp} \cdot g_0 = 130s \cdot 9.81\frac{m}{s^2} = 1275.3\frac{m}{s}$

$v(t_B) = v_{rel} \cdot \ln\left(\frac{m_A}{m_E}\right) - g \cdot t_B \approx 1459\frac{m}{s}$

$$s(t_B) = v_{rel} \cdot t_B - \frac{v_{rel} \cdot m_E \cdot t_B \cdot \ln\left(\frac{m_A}{m_E}\right)}{m_A - m_E} - \frac{g \cdot t_B^2}{2} \approx 7616m$$

$$s_B = \cos(\alpha) \cdot s(t_B) \approx 664m$$

$$h_0 = \sin(\alpha) \cdot s(t_B) \approx 7587m$$

$$s_t = \frac{v(t_B) \cdot \cos(\alpha)}{g} \cdot \left(v(t_B) \cdot \sin(\alpha) + \sqrt{v(t_B)^2 \cdot \sin^2(\alpha) + 2 \cdot g \cdot h_0} \right) + s_b \approx 39km$$